4. Übung Math. I Chemie WS 16/17

1) Gauß-Schema:

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 1 & 1 \\ 2 & 3 & -1 & 1 & -3 \\ -1 & 1 & 1 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 2 \\ 0 & 2 & 1 & 0 & -1 \\ 0 & 5 & -3 & -1 & -7 \\ 0 & 0 & 2 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 2 \\ 0 & 2 & 1 & 0 & -1 \\ 0 & 0 & -\frac{11}{2} & -1 & -\frac{9}{2} \\ 0 & 0 & 1 & 1 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 2 \\ 0 & 2 & 1 & 0 & -1 \\ 0 & 0 & -\frac{11}{2} & -1 & -\frac{9}{2} \\ 0 & 0 & -\frac{9}{2} & 0 & -\frac{9}{2} \end{array}\right) \Rightarrow x_3 = 1 \quad x_4 = -1, \; x_2 = -1, \; x_1 = 1.$$

2a) $B \cdot A$:

$$\left(\begin{array}{ccc|cccc} 2 & 3 & -4 & 4 & -13 & 25 & -20 \\ -1 & 2 & 1 & 4 & 7 & -9 & 12 \\ 3 & 8 & -6 & 13 & -16 & 34 & -23 \end{array}\right) = BA$$

$$\left(\begin{array}{cccc} 1 & -2 & 4 & -3 \\ 2 & 1 & -1 & 2 \\ 1 & 3 & -3 & 5 \end{array}\right)$$

$A^T B$:

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 3 & 15 & -8 \\ -2 & 1 & 3 & 4 & 20 & -9 \\ 4 & -1 & -3 & -6 & -30 & 13 \\ -3 & 2 & 5 & 7 & 35 & -16 \end{array}\right) = A^T B$$

$$\left(\begin{array}{ccc} 2 & 3 & -4 \\ -1 & 2 & 1 \\ 3 & 8 & -6 \end{array}\right)$$

b) $$|B| = \begin{vmatrix} 2 & 3 & -4 \\ -1 & 2 & 1 \\ 3 & 8 & -6 \end{vmatrix} = \begin{vmatrix} 2 & 7 & -2 \\ -1 & 0 & 0 \\ 3 & 14 & -3 \end{vmatrix} = -(-1)\begin{vmatrix} 7 & -2 \\ 14 & -3 \end{vmatrix} = 7$$

$$|C| = \begin{vmatrix} 1 & -1 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 4 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 \\ -1 & 3 & -2 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$z_3 - 2z_1$

c) $B\vec{x} = \vec{b}$:

$$\left(\begin{array}{ccc|c} -1 & 2 & 1 & 1 \\ 2 & 3 & -4 & 1 \\ 3 & 8 & -6 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 2 & 1 & 1 \\ 0 & 7 & -2 & 3 \\ 0 & 14 & -3 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 2 & 1 & 1 \\ 0 & 7 & -2 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$\Rightarrow x_1 = 0, \; x_2 = \frac{3}{7} \; | -x_1 = 1 - 2x_2 \Rightarrow x_1 = -\frac{1}{7}$

$C\vec{x} = \vec{b}$:

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ -1 & 3 & -2 & 1 \\ 2 & -2 & 4 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right) \Rightarrow \text{nicht lösbar}$$

$A\vec{x} = \vec{b}$:

$$\left(\begin{array}{cccc|c} 1 & -2 & 4 & -3 & 1 \\ 2 & 1 & -1 & 2 & 1 \\ 1 & 3 & -5 & 5 & 3 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & 4 & -3 & 1 \\ 0 & 5 & -9 & 8 & -1 \\ 0 & 5 & -9 & 8 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & 4 & -3 & 1 \\ 0 & 5 & -9 & 8 & -1 \\ 0 & 0 & 0 & 0 & 3 \end{array}\right)$$

$\Rightarrow$ nicht lösbar.

d) Da $|B| = 7 \neq 0$ ist ex. B^{-1} und aus:

$Bx = 0 \Rightarrow$ (von links mit B^{-1} multipliziert) $\Rightarrow$

$B^{-1}(Bx) = (B^{-1}B)x = x = B^{-1} \cdot 0 = 0 \Rightarrow$ Es existiert kein $x \neq 0$, mit $Bx = 0$.